A Factor Graph Based Approach in Integration of Doppler Shift with GNSS-INS Positioning System

Md. Ali Hasan[1], M. Humayun Kabir[2], and Wonjae Shin[2]

[1]*Department of AI Convergence Network, Ajou University, Suwon, Republic of Korea*
[2]*Department of Electrical and Computer Engineering, Ajou University, Suwon, Republic of Korea*
E-mail: mdalihasan@ajou.ac.kr, humayun@ajou.ac.kr, wjshin@ajou.ac.kr

Abstract

Global navigation satellite system (GNSS) is generally used for Global positioning. GNSS signal suffers from multipath and non-line-of-sight (NLOS) in dense urban areas. The measurements of GNSS depend on the environment, and it is time-correlated. Filter-based methods typically cannot investigate the time-correlation between historical measurements. Moreover, the filter-based approach is influenced by unexpected outlier measurements. Inertial navigation system (INS) provides navigation information at high frequency which becomes inaccurate over time. In this paper, we propose a factor graph (FG) based GNSS positioning system which integrates the Doppler shift with the GNSS-INS positioning system. FG is a bipartite undirected graph which can consider historical measurement thus, suitable to minimize positioning error in the urban multipath scenario.

I. Introduction

GNSS signal is affected by multipath and non-line-of-sight (NLOS) due to environmental attenuation factors and manmade obstacles which degrades the signal. As a result, a huge error is induced in positioning solutions. However, the inertial navigation system (INS) is less induced on environmental conditions but accumulates error over time. So, the GNSS and INS are complementary, and their fusion is addressed in several research [1]. Extended Kalman filter (EKF) based system achieved the optimal estimation when the first-order Markov chain is considered, and random noise is in Gaussian distribution. However, GNSS measurements are non-Gaussian and highly time-correlated in dense urban areas. As a result, EKF based sensor fusion fails to obtain optimal results in this area. Factor graph optimization (FGO) is a probabilistic graphical model that contains various nodes related to system states and various measurement factors. FGO showed potential in sensor fusion in various challenging GNSS scenarios. As GNSS measurements are time correlated, using FGO we can take the full advantage of historical data. We present the GNSS based positioning as a factor graph considering pseudorange, motion model, INS and Doppler shift as factors. To get the optimal state Levenberg-Marquardt (LM) method can be used.

II. System Model

GNSS system integrates the pseudorange measurement and INS system for more accuracy. Noisy and NLOS GNSS signal has significant pseudorange error which degrades the performance of GNSS-INS system. We can utilize doppler shift measurement to calculate the receiver position in this regard. In FGO all measurements are treated as factors. To improve the accuracy of the GNSS-INS system we introduce doppler shift measurement as a factor. The system model is shown in Figure 1.

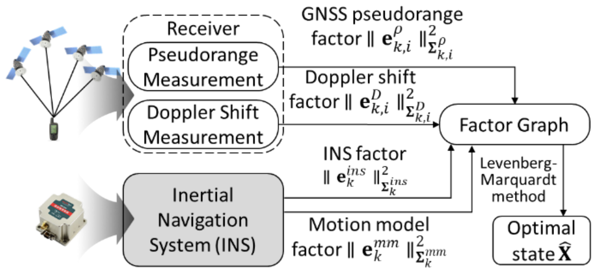

Figure 1:System model for doppler shift integrated GNSS-INS positioning using FGO.

III. Proposed Method

In our research, we have considered 4 factors in the factor graph (FG): INS factor, motion model factor, GNSS pseudorange factor, and Doppler shift factor using Mahalanobis norm. The optimal state of FG can be defined as [2]

$$\hat{\mathbf{X}} = \arg\min_{\mathbf{x}} \left(\sum_j \| h_j(\mathbf{x}_j) - \mathbf{z}_j \|^2_{\Sigma_j} \right), \quad (1)$$

where $h_j(\mathbf{x}_j)$ is the observation function of state $\mathbf{x}_j$ associated with measurement $\mathbf{z}_j$. The FG using Doppler shift integrated GNSS-INS positioning system is shown in Figure 2.

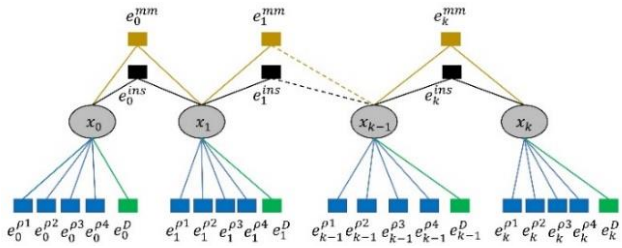

Figure 2: Doppler shift with GNSS-INS positioning system using FGO.

INS factor: The error function for INS acceleration measurements can be formulated as follows

$$\| \mathbf{e}_k^{ins} \|^2_{\boldsymbol{\Sigma}_k^{ins}} = \| \mathbf{x}_k - h^{ins}(\mathbf{x}_{k-1}, \mathbf{A}_k^{ecef}) \|^2_{\boldsymbol{\Sigma}_k^{ins}}, \quad (2)$$

where $h^{ins}(\mathbf{x}_{k-1}, \mathbf{A}_k^{ecef}) = \begin{bmatrix} v_{k-1,r,x}^{ecef} + a_{k,x}^{ecef}.\Delta t \\ v_{k-1,r,y}^{ecef} + a_{k,y}^{ecef}.\Delta t \\ v_{k-1,r,z}^{ecef}.+a_{k,z}^{ecef}\Delta t \end{bmatrix}$, $\mathbf{x}_k$ is the position state of the receiver at epoch k, $\mathbf{A}_k^{ecef}$ is the acceleration measurement form inertial measurement unit (IMU) in global coordinate, $\boldsymbol{\Sigma}_k^{ins}$ is the error covariance.

Motion model factor: The error function of the motion model yields

$$\| \mathbf{e}_k^{mm} \|^2_{\boldsymbol{\Sigma}_k^{mm}} = \| \mathbf{x}_k - h^{mm}(\mathbf{x}_{k-1}) \|^2_{\Sigma_k^{mm}}, \quad (3)$$

where $h^{mm}(\mathbf{x}_{k-1}) = \begin{bmatrix} x_{k-1,r}^{ecef} + v_{k-1,r,x}^{ecef}.\Delta t \\ y_{k-1,r}^{ecef} + v_{k-1,r,y}^{ecef}.\Delta t \\ z_{k-1,r}^{ecef} + v_{k-1,r,z}^{ecef}.\Delta t \\ \boldsymbol{B}_{k-1,r}^{ins\ T} \end{bmatrix}$, $\boldsymbol{B}_{k-1,r}^{ins}$ is the accelerometer bias in INS frame.

GNSS pseudorange factor: The error function for pseudoarange measurement is

$$\| \mathbf{e}_{k,i}^{\rho} \|^2_{\boldsymbol{\Sigma}_{k,i}^{\rho}} = \| \rho_{k,i} - h^{GNSS}(\mathbf{SV}_{k,i}, \mathbf{X}_{k,r}^{ecef}, \delta_{k,r}^{clock}) \|^2_{\boldsymbol{\Sigma}_{k,i}^{\rho}}, \quad (4)$$

where $h^{GNSS,TC}(\mathbf{SV}_{k,i}, \mathbf{X}_{k,r}^{ecef}, \boldsymbol{\delta}_{k,r}^{clock}) = \begin{bmatrix} \| \boldsymbol{SV}_{k,1} - \boldsymbol{X}_{k,r}^{ecef} \| + \boldsymbol{\delta}_{k,r}^{clock} \\ \| \boldsymbol{SV}_{k,2} - \boldsymbol{X}_{k,r}^{ecef} \| + \boldsymbol{\delta}_{k,r}^{clock} \\ \cdots \\ \| \boldsymbol{SV}_{k,i} - \boldsymbol{X}_{k,r}^{ecef} \| + \boldsymbol{\delta}_{k,r}^{clock} \\ \cdots \\ \| \boldsymbol{SV}_{k,N} - \boldsymbol{X}_{k,r}^{ecef} \| + \boldsymbol{\delta}_{k,r}^{clock} \end{bmatrix}$,

$\mathbf{SV}_{k,i}$ is the i^{th} satellite position at epoch k, $\boldsymbol{\delta}_{k,r}^{clock}$ is the receiver clock bias.

Doppler shift factor: The error function for doppler shift measurement is

$$\| \mathbf{e}_{k,i}^{D} \|^2_{\boldsymbol{\Sigma}_{k,i}^{D}} = \| \lambda_i D_{k,i} - h^{D}(\mathbf{SV}_{k,i}, \mathbf{x}_k, \dot{\delta}_{k,r}^{clock}) \|^2_{\boldsymbol{\Sigma}_{k,i}^{D}}, \quad (5)$$

where $h^D(x_k^s, x_k, \dot{\delta}_{k,r}^{clock}) = \left[v.\frac{\mathbf{SV}_{k,i} - \mathbf{x}_k}{\|\mathbf{SV}_{k,i} - \mathbf{x}_k\|} \right]$, $D_{k,i}$ and λ_i is the wavelength and doppler shift for i^{th} satellite respectively at epoch k, v is the relative velocity, $\dot{\delta}_{k,r}^{clock}$ receiver clock drift. The optimal state $\mathbf{X} = \{\mathbf{x}_1, \mathbf{x}_2, \ldots\ldots, \mathbf{x}_k, ..\}$ is obtained by solving the following equation:

$$\mathbf{X}^* = \arg\min \sum_{i,k} \| \mathbf{e}_{k,i}^{\rho} \|^2_{\boldsymbol{\Sigma}_{k,i}^{\rho}} + \| \mathbf{e}_{k,i}^{D} \|^2_{\boldsymbol{\Sigma}_{k,i}^{D}} + \| \mathbf{e}_k^{mm} \|^2_{\boldsymbol{\Sigma}_k^{mm}} + \| \mathbf{e}_k^{ins} \|^2_{\boldsymbol{\Sigma}_k^{ins}} \quad (6)$$

IV. Conclusion

Nowadays positioning is essential nowadays in many tracking and control system. GNSS system is mostly used for global positioning. In this paper, we have realized FGO method considering four different factors. To overcome the limitations of present GNSS-INS positioning system, we have introduced doppler shift as a factor in factor graph.

ACKNOWLEDGEMENT

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